

Circling the Square: P.H. Smith and His Chart

Ancient Greek mathematicians unsuccessfully spent a long time trying to square the circle. Sadly, π is irrational, but one American went the other way around, "circling" a square.

While we are all familiar with Smith charts, which represent the circles of normalized impedances and admittances within a unitary circle on the reflection coefficient complex plane, few may know that the original Smith chart was square. The earliest Smith chart was devised by Philip Hagar Smith in 1931 while trying to find a guick graphical way to compute the matching network for a large transatlantic communications antenna.¹⁻³ The first draft was square and drawn on the normalized impedance plane, with circumferences showing the impedance transport along the line. Distances from the load are given by the radial curves emerging from the central matched impedance point and marked in wavelengths from 0 to 0.5, as shown in Figure 1.

This was far from perfect because the reflection coefficient could not be displayed graphically and impedances might very well be outside the chart. The second try in 1936 was better. In polar coordinates, the plane had VSWR as a radial coordinate and the distance in wavelengths from the VSWR minimum as an angular coordinate. This was nice since VSWR was directly measured on the line, but it was a nightmare to draw lines at the constant real and imaginary parts of the normalized impedance, which were not circles, as shown in Figure 2.

The third time was the charm. Using the reflection coefficient (modulus and phase) to define the plane, the curves at constant real and imaginary parts of the normalized impedance become circles with an easy analytic formula.⁴ This chart, developed in 1939, is still in use today. It is worth noting that Tosaku Mizuhashi independently obtained the same chart as evidenced by a Japanese paper, but this did not reach Western researchers until too late.^{2,5}

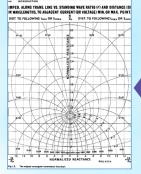
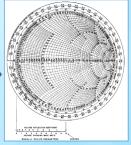


Fig. 1 The 1931 square Smith chart.1

Fig. 2 The 1936 circular Smith chart.1



References

- 1. P. H. Smith, "Electronic Applications of the Smith Analysis," New York: McGraw-Hill, 1969
- 2. S. Maddio, G. Pelosi and S. Selleri, "A Brief History of the Smith Chart," IEEE HISTELCON, Florence, Italy, 2023, pp. 39-41.
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- 5. T. Mizuhashi, "Theory of Four-terminal Impedance Transformation Circuit and Matching Circuit," J. Inst. Elec. Commun. Engr. Japan, 20, 1937, pp. 1053–1058 (in Japanese).